



PART 1

Propagation Characteristics of Wireless Channels

Lecture 1.2

Attenuation models



Power units - decibel

- ➔ **Decibel (dB):** logarithmic unit of intensity used to indicate *power* lost or gained between two signals
- ➔ **Named after Alexander Graham Bell.**

$$10 \log(P_1 / P_2)$$

$$P_A = 1 \text{ Watt}$$

$$P_B = 50 \text{ milliwatt}$$

$$\rightarrow P_A = 13 \text{ dB greater than } P_B$$



Decibels - dBm

→ dBm = absolute value (reference= 1mW)

$$\Rightarrow \text{Power in dBm} = 10 \log(\text{power}/1\text{mW})$$

$$\Rightarrow \text{Power in dBW} = 10 \log(\text{power}/1\text{W})$$

» Not much used by us

$$1\text{dBW}=30\text{dBm}$$

→ Examples

$$\Rightarrow 10 \text{ mW} = 10 \log_{10}(0.01/0.001) = 10 \text{ dBm}$$

$$\Rightarrow 10 \mu\text{W} = 10 \log_{10}(0.00001/0.001) = -20 \text{ dBm}$$

$$\Rightarrow 26 \text{ dBm} = \underline{\hspace{1cm}} \quad 2\text{W} = \underline{\hspace{1cm}} \text{ dBm?}$$

$$\Rightarrow \text{S/N ratio} = -3\text{dB} \rightarrow \text{S} = \underline{\hspace{1cm}} \times \text{N?}$$

→ Properties & conversions

$$\Rightarrow \text{dBm} = 10 \log_{10}(P (\text{W}) / 1 \text{ mW}) = P (\text{dB}) + 30 \text{ dBm}$$

$$\Rightarrow P1 * P2 (\text{dBm}) = P1 (\text{dBm}) + P2 (\text{dB})$$

$$\begin{aligned} P1 * P2 (\text{dBm}) &= 10 \log_{10}(P1 * P2 (\text{W}) / 0.001) = \\ &10 \log_{10}(P1 / 0.001) + 10 \log_{10} P2 = P1 (\text{dBm}) + P2 (\text{dB}) \end{aligned}$$



Computation with dB

→ Transmit power

⇒ Measured in dBm

→ Es. 33 dBm

→ Receive Power

⇒ Measured in dBm

→ Es. -10 dBm

→ Path Loss

⇒ Receive power / transmit power

⇒ Measured in dB

⇒ Loss (dB) = transmit (dBm) – receive (dBm)

→ Es. 43 dB = attenuation by factor 20.000



Attenuation model for LOS

→ Direct path between transmitter and receiver

- ⇒ unobstructed Line-Of-Sight (LOS)
- ⇒ Radio signal behaves like light in free space (straight line)

→ Receive power:

- ⇒ In absence of obstacles, received power follows inverse square law

$$P_r(d) \propto d^{-2}$$

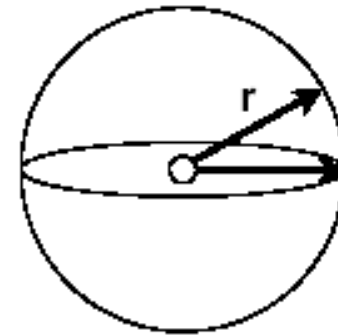
→ (d = distance between sender and receiver)



free space model – ideal antennas

→ Isotropic (omnidirectional) tx antenna in free space

- ⇒ Transmitted power: P_t
- ⇒ Power attenuation P_a at distance d :
down with sphere superficies



$$P_a(d) = \frac{P_t}{4\pi d^2}$$

→ Power received by isotropic rx antenna

- ⇒ Planar wave
- ⇒ A_e = Effective Area

$$P_r(d) = P_a(d) A_e$$

$$A_e = \frac{\lambda^2}{4\pi}$$

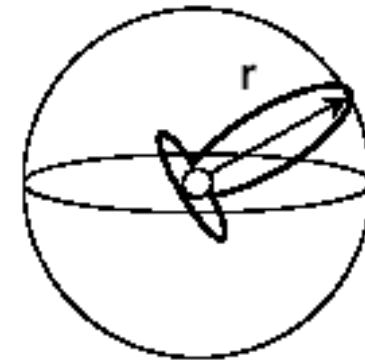


free space model – real antennas

→ Non isotropic tx antenna

⇒ Antenna gain G_t

$$P_a(d) = \frac{G_t P_t}{4\pi d^2}$$



→ Non isotropic rx antenna

⇒ Antenna gain G_r

$$P_r(d) = P_a(d) G_r A_e = \frac{P_t G_r}{4\pi d^2} G_r \frac{\lambda^2}{4\pi}$$



Friis Free-Space Model

summarizing all previous considerations

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \quad d > 0$$

→ P_t = transmitter power

⇒ (W or mW)

→ G_t = transmitter antenna gain

→ G_r = receiver antenna gain

⇒ (dimensionless)

→ $\lambda = c/f$ = RF wavelength (m)

⇒ c = speed of light (3×10^8 m/s)

⇒ f = RF frequency (Hz)

→ $P_t G_t$ = Equivalent Isotropic Radiated Power (EIRP)

→ L = other system losses (hardware)

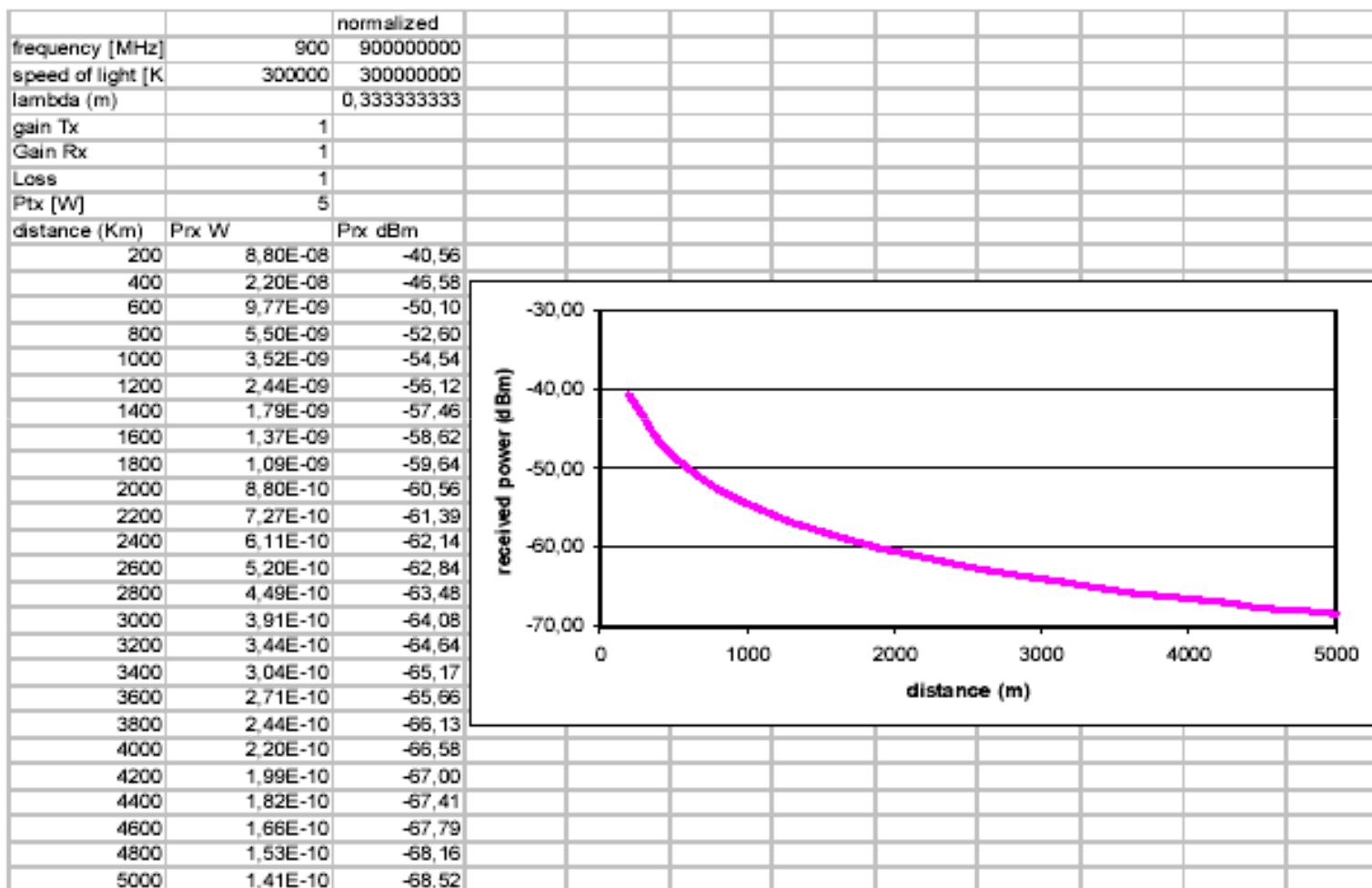
⇒ $L \geq 1$ (dimensionless)

→ d = distance between transmitter and receiver (m)

$$P_r(d) [dBm] = P_t [dBm] + 10 \log_{10} G_t + 10 \log_{10} G_r + 20 \log_{10} \lambda - 20 \log_{10} (4\pi) - 20 \log_{10} d - 10 \log_{10} L$$



Example





Path Loss; Free space loss

both taken as positive values (in dB)

→ Path loss

⇒ Normalized versus transmitted power

$$\frac{P_r(d)}{P_t} = \frac{G_t G_r}{L} \cdot \left(\frac{\lambda}{4\pi d} \right)^2$$

$$PL(d)(dB) = 20 \log_{10} d - 10 \log_{10} \frac{G_t G_r}{L} - 20 \log_{10} \frac{\lambda}{4\pi}$$

→ Free space loss

⇒ Part due to attenuation in free space, only

$$L_{free}(d) = \left(\frac{\lambda}{4\pi d} \right)^{-2}$$

$$L_{free}(d)(dB) = -20 \log \left[\frac{\lambda}{4\pi d} \right] = -20 \log \left[\frac{c/f}{4\pi d} \right]$$



More realistic propagation models

→ Inverse square power law

- ⇒ Way too optimistic (ideal case)
- ⇒ Real world: η -th power law

$$P_r(d) \propto d^{-\eta}$$

- ⇒ with η ranging up to as much as $\eta=7$
 - If tough environment (e.g., lots of foliage),
- ⇒ typical values:
 - $\eta=2$ for small distances (20 dB/decade)
 - $\eta=3$ to $\eta=4$ (40 dB/decade) for mobile telephone distances
- ⇒ η higher in cities and urban areas; η lower in suburban or rural areas.



Realistic scenarios

→ obstructions between the transmitter and receiver

- ⇒ reflection, diffraction, scattering
- ⇒ Propagation strongly influenced by environment (building characteristics, vegetation density, terrain variation)
- ⇒ Perfect conductors reflect waves; nonconductors absorb some energy

→ wave traverses multiple paths

- ⇒ Radio waves arrive at receiver from different directions and with different time delays

→ Resultant signal at receiving antenna is vector addition of incoming signals

- ⇒ signals can add constructively (resultant signal has large power) or destructively (resultant signal has small power) depending on relative phases

Software tools needed to analyze complex specific scenarios (ray-tracing)



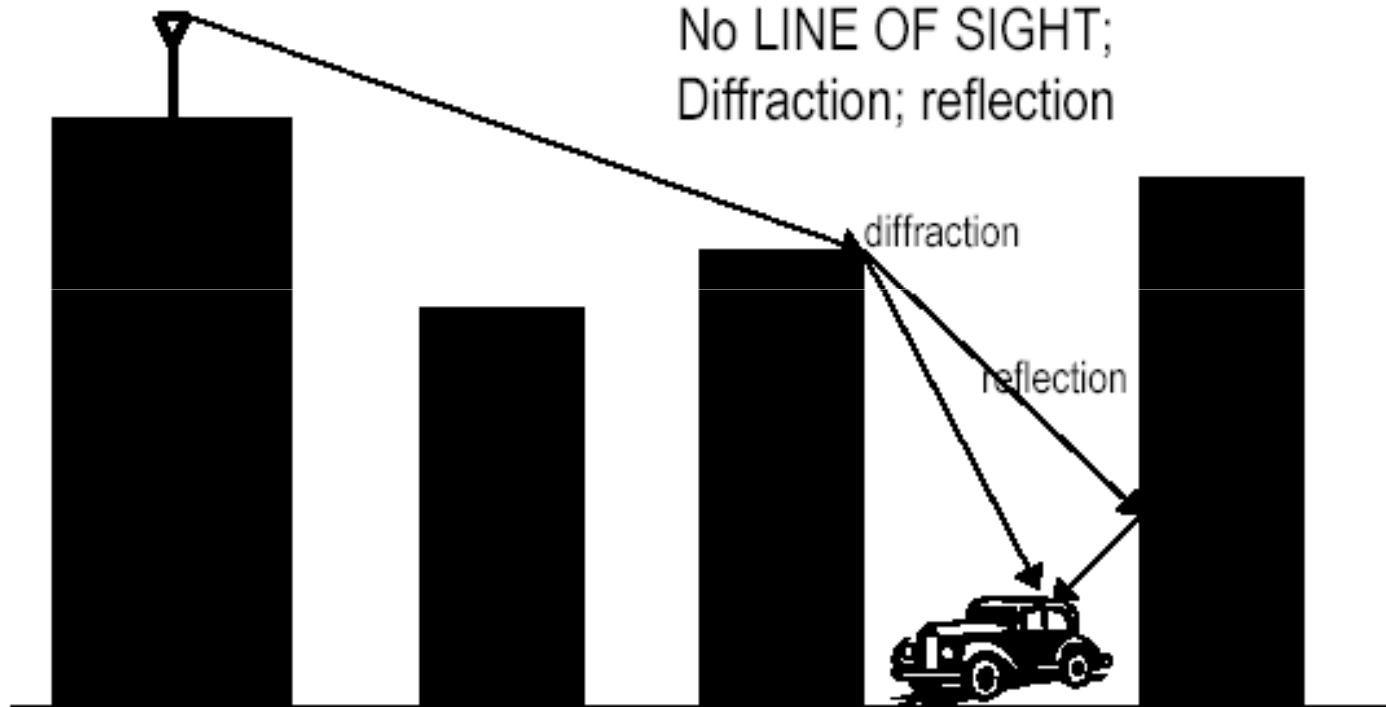
Example scenarios:

LOS path non necessarily existing (and unique)

Example: city with large buildings;

No LINE OF SIGHT;

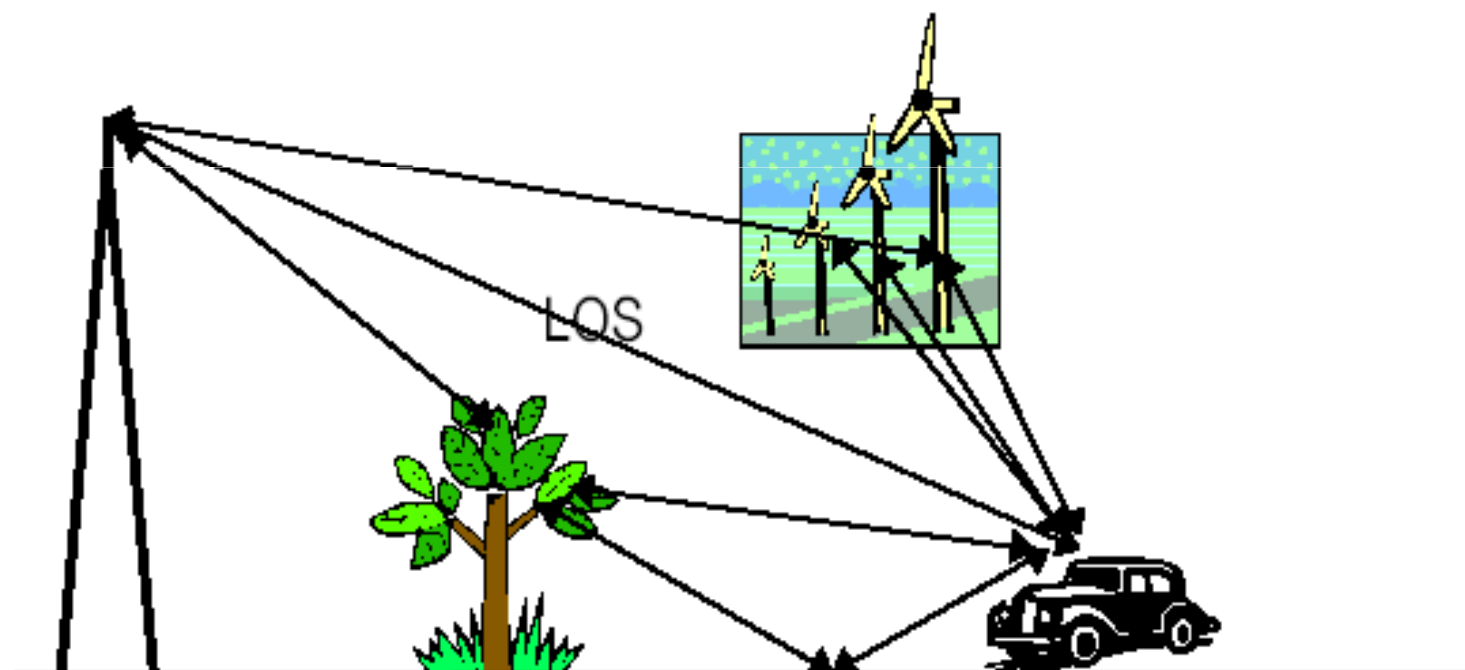
Diffraction; reflection





Example scenarios

LINE OF SIGHT +
Diffraction, reflection, scattering

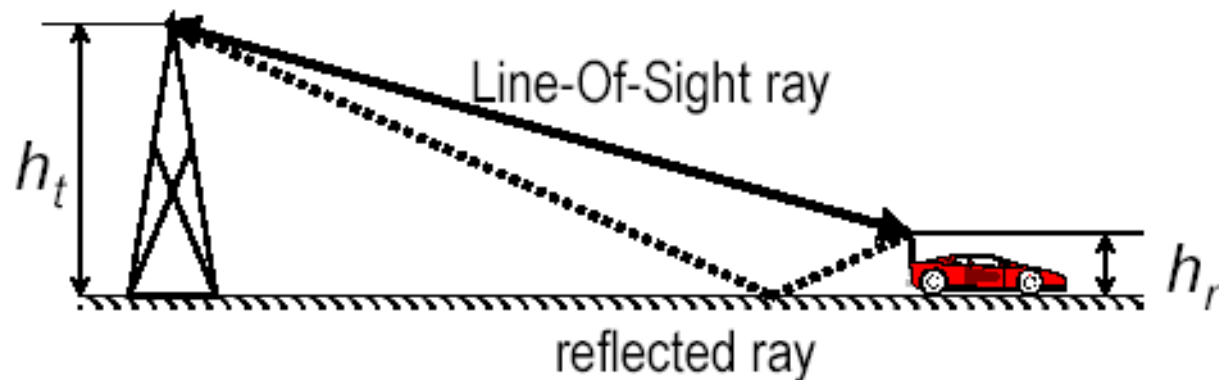




Two-Ray Ground Propagation Model

→ Theoretical foundation for $\eta=4$

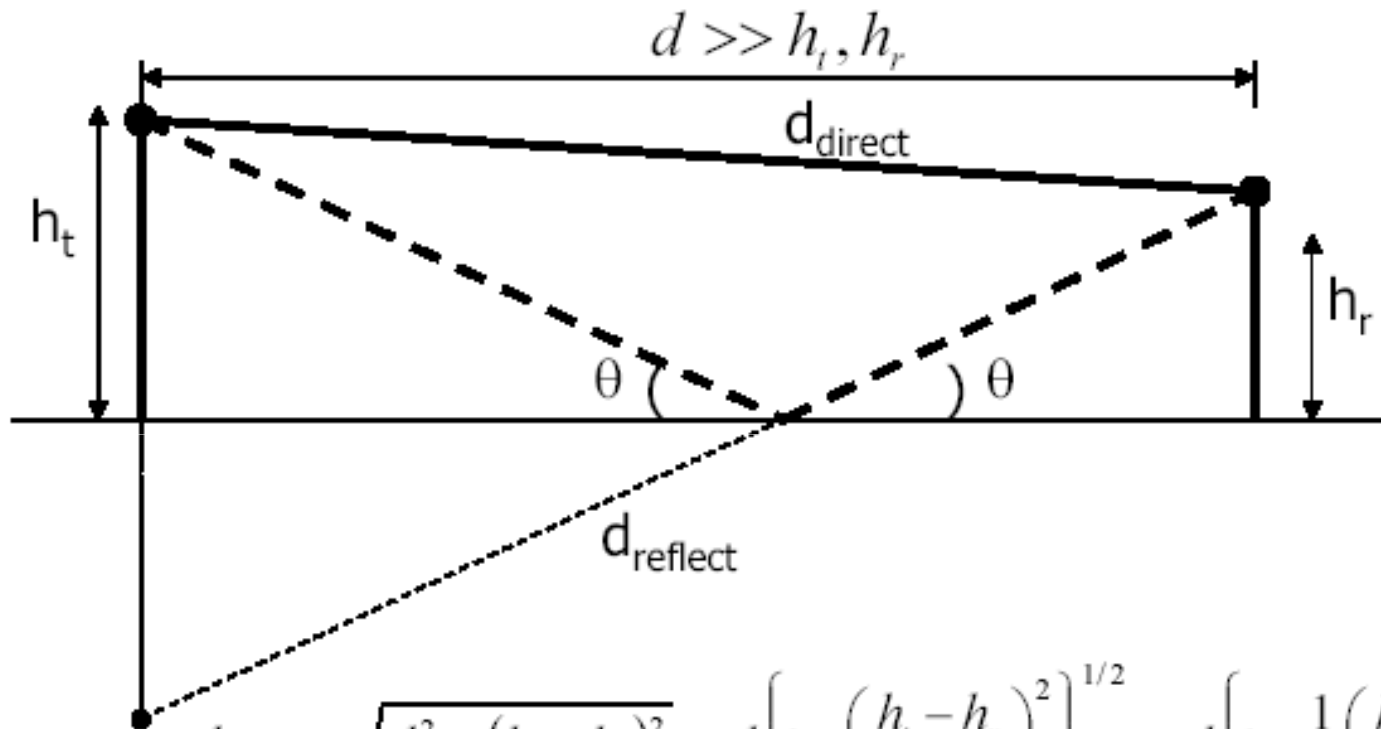
- ⇒ Two-ray model assumes one direct LOS path and one reflection path reach receiver with significant power
- ⇒ Easy to solve



Transmit and receive antennas at different height (in general)



Two-ray model – geometry



$$d_{\text{direct}} = \sqrt{d^2 + (h_t - h_r)^2} = d \left\{ 1 + \left(\frac{h_t - h_r}{d} \right)^2 \right\}^{1/2} \approx d \left\{ 1 + \frac{1}{2} \left(\frac{h_t - h_r}{d} \right)^2 \right\}$$

$$d_{\text{reflect}} = \sqrt{d^2 + (h_t + h_r)^2} = d \left\{ 1 + \left(\frac{h_t + h_r}{d} \right)^2 \right\}^{1/2} \approx d \left\{ 1 + \frac{1}{2} \left(\frac{h_t + h_r}{d} \right)^2 \right\}$$



Two ray model – path analysis

$$d_{\text{reflect}} - d_{\text{direct}} \approx d \left\{ 1 + \frac{1}{2} \left(\frac{h_t + h_r}{d} \right)^2 \right\} - d \left\{ 1 + \frac{1}{2} \left(\frac{h_t - h_r}{d} \right)^2 \right\} = 2 \frac{h_t h_r}{d}$$

→ EM waves travel for different distance

⇒ Sum up with different phase!

$$\text{direct ray} \propto A \cos \left[2\pi f \left(t - \frac{d_{\text{direct}}}{c} \right) \right]$$

$$\text{reflect ray} \propto B \cos \left[2\pi f \left(t - \frac{d_{\text{reflect}}}{c} \right) \right]$$

⇒ A = attenuation along direct path

⇒ B = attenuation along reflected path (reflection not ideal, in general)



Two ray model – vector sum

→ Phase difference

$$\Delta\varphi = 2\pi f \frac{\Delta d}{c} = \frac{2\pi}{\lambda} \Delta d = \frac{4\pi h_t h_r}{\lambda d}$$

→ Received power

⇒ Assume $d_{\text{direct}} \sim d_{\text{reflect}}$

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \cdot |1 + \rho e^{j\Delta\varphi}|$$

→ Assume ideal reflection ($\rho = -1$)

$$|1 - e^{j\Delta\varphi}| = 2 - 2\cos(\Delta\varphi) = 4\sin^2(\Delta\varphi / 2)$$

$$P_r(d) = \frac{P_t G_t G_r}{L} \cdot \left(\frac{\lambda}{4\pi d}\right)^2 \cdot 4\sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$



Two ray model - conclusion

→ Typical values:

$$\begin{aligned} \Rightarrow h_t &\sim \text{few tens of m} \\ \Rightarrow h_r &\sim \text{couple of meters} \\ \Rightarrow \lambda &\sim \text{few tens of cm} \\ \Rightarrow d &\sim \text{hundred meters – few km} \end{aligned} \quad \frac{2\pi h_t h_r}{\lambda d} \approx \text{small} \Rightarrow \sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right) \approx \left(\frac{2\pi h_t h_r}{\lambda d}\right)^2$$

$$P_r(d) \approx \frac{P_t G_t G_r}{L} \cdot \left(\frac{\lambda}{4\pi d}\right)^2 \cdot 4 \left(\frac{2\pi h_t h_r}{\lambda d}\right)^2 = \frac{P_t G_t G_r}{L} \cdot \frac{h_t^2 h_r^2}{d^4}$$

***i.e. attenuation follows a 40 dB/decade rule!
Versus 20 dB/decade of the free-space model***

$$P_r(d) \propto d^{-4}$$



Design notes

→ Typical assumptions for initial system developement

⇒ $\eta=2$ power law attenuation for small distances

→ Free space model

⇒ $\eta=4$ power law attenuation for large distances

→ LOS + reflected ray model



Empirical models

→ Consider specific scenarios

- ⇒ Urban area (large-medium-small city), rural area
- ⇒ Models generated by combining most likely ray traces (LOS, reflected, diffracted, scattered)
- ⇒ Based on large amount of empirical measurements

→ Account for parameters

- ⇒ Frequency; antenna heights; distance

→ Account for correction factors

- ⇒ (diffraction due to mountains, lakes, road shapes, hills, etc)

First model: Okumura, 1968

VERY complex due to many specific correction factors!

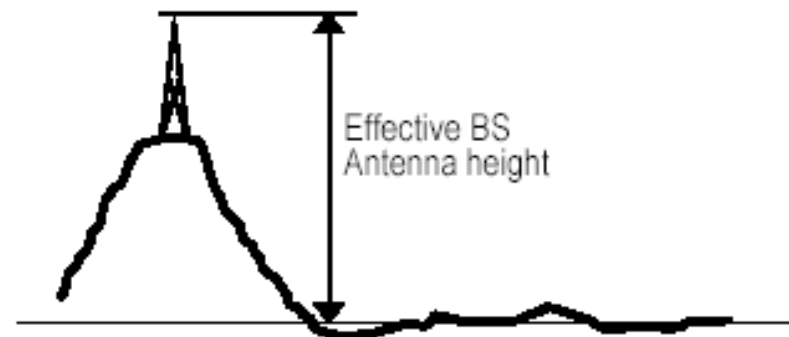


Okumura-Hata model

- **Hata (1980): very simple model to fit Okumura results**
- **Provide formulas to evaluate path loss versus distance for various scenarios**
 - ⇒ Large cities; Small and medium cities; Rural areas
 - ⇒ Limit: $d \geq 1\text{km}$

Parameters:

- f = carrier frequency (MHz)
- d = distance BS → MS (Km)
- h_{bs} = (effective) height of base station antenna (m)
- h_{ms} = height of mobile antenna (m)





Okumura-Hata: urban area

$$L_{path}(dB) = 69.55 + 26.16 \log_{10} f + \\ + (44.9 - 6.55 \log_{10} h_{bs}) \log_{10} d + \\ - 13.82 \log_{10} h_{bs} - a(h_{ms})$$

→ $a(h_{ms})$ = correction factor to differentiate large from medium-small cities;

→ depends on MS antenna height

large cities : $a(h_{ms}) = 3.2 [\log_{10} (11.75 h_{ms})]^2 - 4.97 \quad f \geq 400 MHz$

small - med cities : $a(h_{ms}) = [1.1 \log_{10} f - 0.7] h_{ms} - [1.56 \log_{10} f - 0.8]$



Okumura-Hata: suburban & rural areas

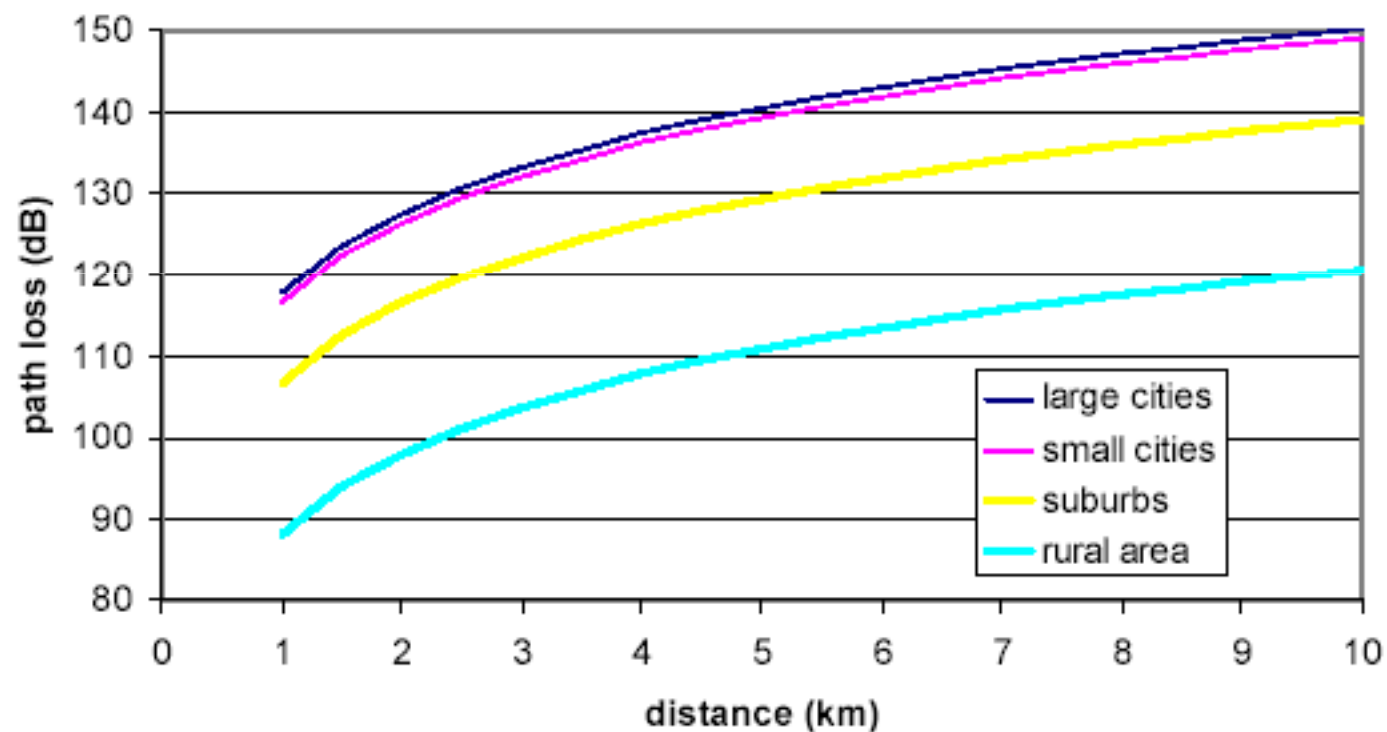
→ Start from path loss L_p computed for small and medium cities

$$\text{suburban : } L_{path}(dB) = L_p - 2 \left[\log_{10} \frac{f}{28} \right]^2 - 5.4$$

$$\text{rural : } L_{path}(dB) = L_p - 4.78 [\log_{10} f]^2 + 18.33 \log_{10} f - 40.94$$



Okumura-Hata: examples



$F=900\text{MHz}$, $h_{bs}=80\text{m}$, $h_{ms}=3\text{m}$



Other empirical models

→ Lee's model

- ⇒ Use at 900MHz
- ⇒ For distances > 1km
- ⇒ Based on measurements taken in three cities (including Philadelphia)
- ⇒ More complex than Okumura-Hata

→ Walfish-Ikegami model

- ⇒ For frequency range 800-2000 MHz
- ⇒ Valid for microcellular distances (20m – 5 km)
- ⇒ Adopted by European Cooperation in the field of Scientific and Technical (COST) research as reference model for 3G systems

→ Indoor propagation models

- ⇒ Include attenuation factors due to building penetration
- ⇒ Account for number of walls, floors, reflection loss, etc
- ⇒ Based on zones (large zone, middle zone, small zone, microzone)



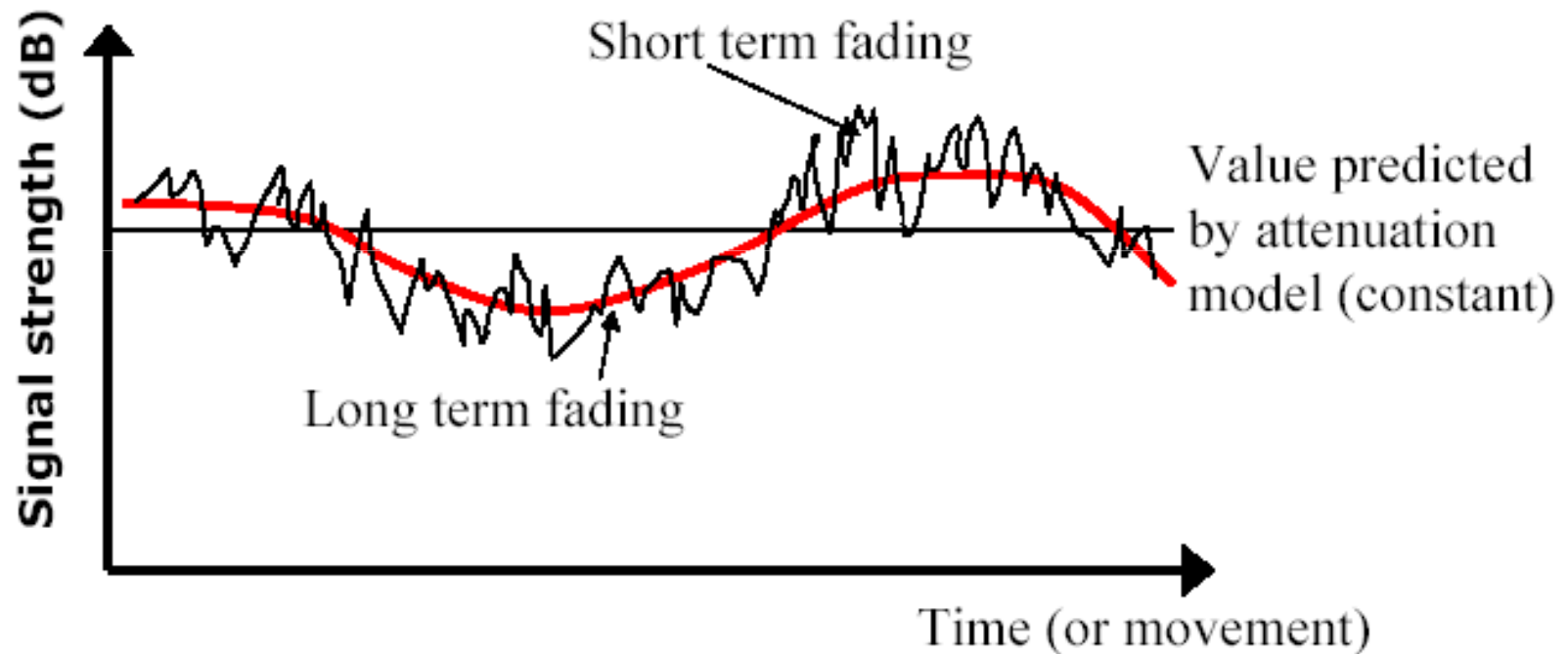
PART 1

Propagation Characteristics of Wireless Channels

Lecture 1.3 fading models



Statistical nature of received power





Multipath: short term fading

→ Short-term fading

- ⇒ Also: multipath fading
- ⇒ Also: small-scale fading
- ⇒ Also: fading

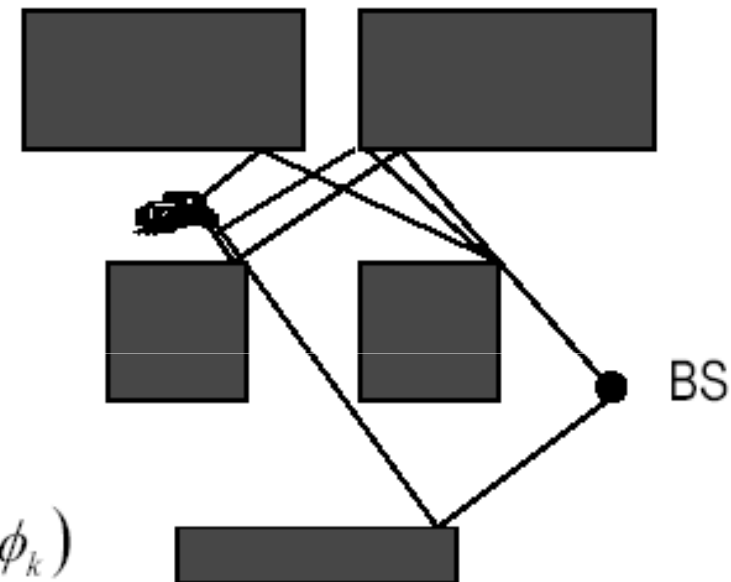
→ Generated by superposition of same signal travelling along many paths

→ Received signal:

$$e_r(t) = \sum_{k=1}^N a_k \cos(2\pi f_0 t + \phi_k)$$

Phase depends on path length

- f_0 =carrier frequency
- N =number of paths
- a_k, ϕ_k =amplitude & phase of component k



Multipath fading consequences

- ⇒ Small movements of tx or rx (order of $\frac{1}{2}\lambda$)
 - change interference pattern
 - drastic fluctuations in signal strength due to constructive/destructive interference
 - 15-20 cm for 900 MHz



Multipath analysis

$$e_r(t) = \sum_{k=1}^N a_k \cos(2\pi f_0 t + \phi_k) =$$

$$\begin{aligned} \text{recall that : } \cos(2\pi f_0 t + \phi_k) &= \\ &= \cos(2\pi f_0 t) \cos(\phi_k) - \sin(2\pi f_0 t) \sin(\phi_k) \end{aligned}$$

$$= \cos(2\pi f_0 t) \sum_{k=1}^N a_k \cos \phi_k - \sin(2\pi f_0 t) \sum_{k=1}^N a_k \sin \phi_k =$$

$$= X \cos(2\pi f_0 t) - Y \sin(2\pi f_0 t)$$

In the assumptions:

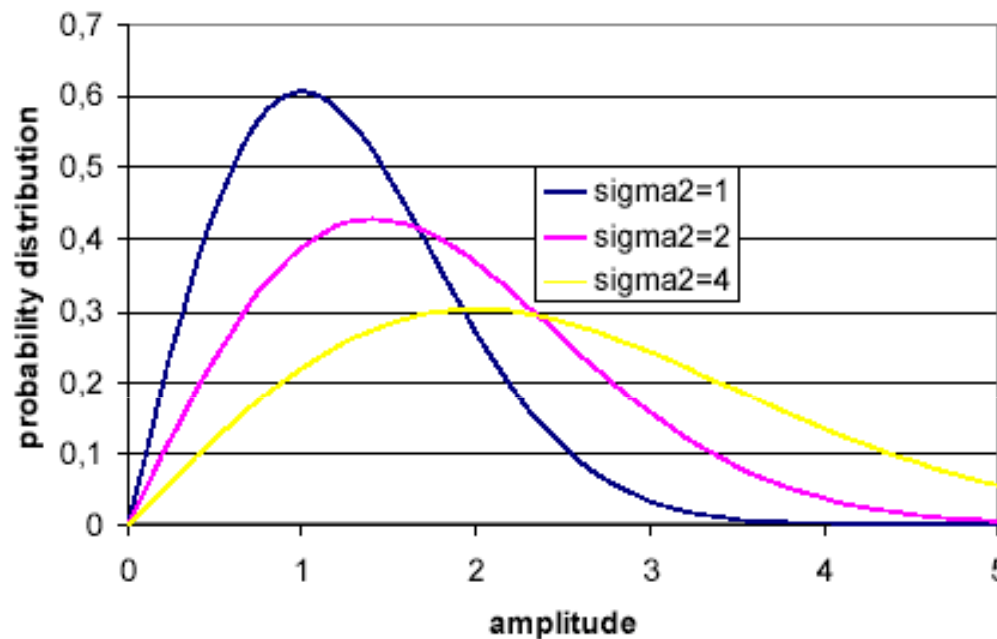
- N large (many paths)
- ϕ_k uniformly distributed in $(0, 2\pi)$
- a_k comparable (no privileged path such as LOS)

X, Y are gaussian, identically distributed random variables

$$\text{Signal envelope: } \sqrt{X^2 + Y^2} \quad \text{Rayleigh distribution}$$



Rayleigh distribution



σ^2 = variance of X and Y
Gaussian r.v.

$$\begin{aligned} f_a(x) &= \\ &= \Pr(x \leq a < x + dx) = \\ &= \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} \end{aligned}$$

$$E[a] = \int_0^{\infty} x \cdot \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} = \sigma \sqrt{\frac{\pi}{2}} = 1.253\sigma$$

$$Var[a] = \int_0^{\infty} x^2 \cdot \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} - \sigma^2 \frac{\pi}{2} = \sigma^2 \left(2 - \frac{\pi}{2} \right) = 0.4292\sigma^2$$



Signal power

amplitude: $a = \sqrt{X^2 + Y^2}$: rayleigh distribution

power: $p = a^2 = X^2 + Y^2$: exponential distribution

→ Average power:

⇒ $2\sigma^2$ (average over time)

→ Instantaneous power

⇒ Random variable, with:

→ probability density function:

$$f_p(x) = \frac{1}{2\sigma^2} e^{-x/2\sigma^2}$$

→ Probability distribution function:

$$F_p(x) = 1 - e^{-x/2\sigma^2}$$



Outage probability

→ **Probability that received power is lower than a given threshold**

⇒ Below which signal cannot be correctly received

→ **Average received power $P_0 (= 2\sigma^2)$**

→ **Minimum power threshold γ**

outage probability : $\Pr(p \leq \gamma) = F_p(\gamma) = 1 - e^{-\gamma/P_0}$

Example 1:

average power = $100 \mu\text{W}$;

lower threshold = $15 \mu\text{W}$;

Outage probability = $1 - \exp(-15/100) = 13,9\%$

Example 2:

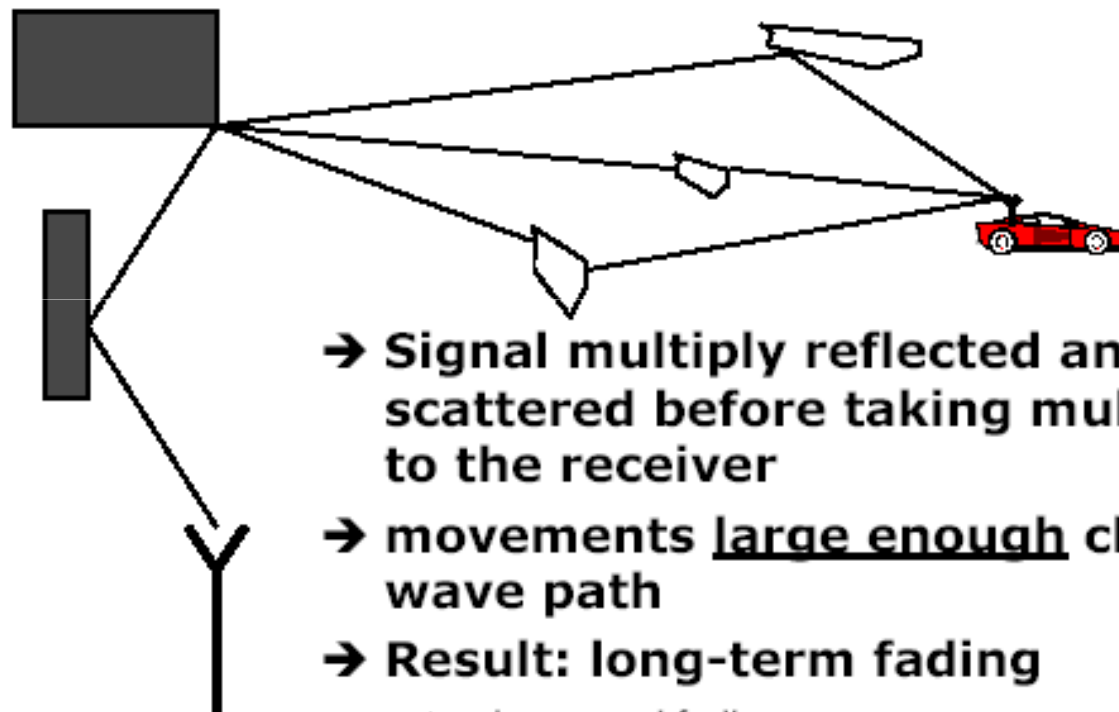
average power = -13 dBm ;

lower threshold = -30 dBm ;

Outage probability = $1 - \exp(-1/50) = 1,98\%$



Long-term fading



→ Signal multiply reflected and/or scattered before taking multiple paths to the receiver

→ movements large enough change wave path

→ Result: long-term fading

⇒ = lognormal fading

⇒ = shadowing



Long-term fading statistics lognormal distribution

$$P_r(d)(dB) = 10 \log_{10} P_r(d_o) + 10\eta \log_{10} \left(\frac{d_o}{d} \right) + Y$$

→ **Y = 0 mean gaussian r.v.**
with standard deviation σ_{dB} dB

→ **Probability distribution (in dB):**

$$f_Y(p_{dB}) = \frac{1}{\sqrt{2\pi} \cdot \sigma_{dB}} e^{-\frac{(p_{dB} - P_{av})^2}{2\sigma_{dB}^2}}$$



Long-term fading and attenuation plot

attenuation: $\eta=4$ after 100m; $\eta=2$ before 100m

